An MPEC Formulation for Parameter Identification of Complementarity Systems

S. Berard J.C. Trinkle

Department of Computer Science
Rensselaer Polytechnic Institute

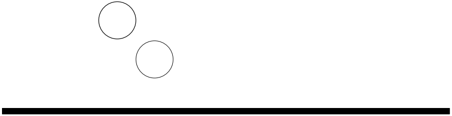
May 30, 2008

Problem Statement

Using noisy observations of a dynamical multi-rigid-body system, determine the parameters of a given mixed complementarity problem dynamics model that best predicts the observation.

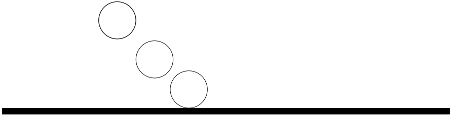
Problem Statement

Using noisy observations of a dynamical multi-rigid-body system, determine the parameters of a given mixed complementarity problem dynamics model that best predicts the observation.



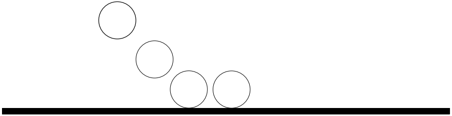
Problem Statement

Using noisy observations of a dynamical multi-rigid-body system, determine the parameters of a given mixed complementarity problem dynamics model that best predicts the observation.



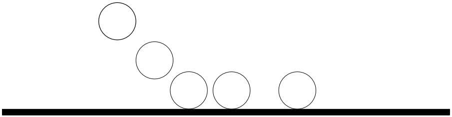
Problem Statement

Using noisy observations of a dynamical multi-rigid-body system, determine the parameters of a given mixed complementarity problem dynamics model that best predicts the observation.



Problem Statement

Using noisy observations of a dynamical multi-rigid-body system, determine the parameters of a given mixed complementarity problem dynamics model that best predicts the observation.



Coefficient of friction $= 0.2$

Complementarity Problem

Let $\mathbf{u} \in \mathbb{R}^{n_1}$, $\mathbf{v} \in \mathbb{R}^{n_2}$ and let $g : \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \longrightarrow \mathbb{R}^{n_1}$, $f : \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \longrightarrow \mathbb{R}^{n_2}$ be two vector functions and the notation $0 \leq \mathbf{x} \perp \mathbf{y} \geq 0$ imply that $\mathbf{x}$ is orthogonal to $\mathbf{y}$ and each component of each vector is non-negative.

Definition

The mixed complementarity problem is to find $\mathbf{u}$ and $\mathbf{v}$ satisfying

$$g(\mathbf{u}, \mathbf{v}) = 0, \quad \mathbf{u} \text{ free}$$
$$0 \leq \mathbf{v} \perp f(\mathbf{u}, \mathbf{v}) \geq 0$$

Orthogonality

$$\mathbf{x} = \begin{bmatrix} 0 \\ 0 \\ + \\ \vdots \end{bmatrix} \begin{bmatrix} + \\ + \\ 0 \\ \vdots \end{bmatrix} = \mathbf{y}$$

Instantaneous Dynamics Model

$\mathbf{M}$: Inertia matrix
$\mathbf{W}_{\mathrm{n}}$: Maps normal forces to body frame
$\mathbf{W}_{\mathrm{f}}$: Maps friction forces to body frame
$\mathbf{q}$: Configuration
$\boldsymbol{\nu}$: Velocity

$\boldsymbol{\lambda}_{\mathrm{vp}}$: Velocity product forces
$\boldsymbol{\lambda}_{\mathrm{app}}$: Applied forces
$\boldsymbol{\lambda}_{\mathrm{n}}$: Magnitude of normal forces
$\boldsymbol{\lambda}_{\mathrm{n}}$: Magnitude of frictional forces

- Newton-Euler Equations:
$\mathbf{M}(\mathbf{q})\dot{\boldsymbol{\nu}} = \mathbf{W}_{\mathrm{n}}\boldsymbol{\lambda}_{\mathrm{n}} + \mathbf{W}_{\mathrm{f}}\boldsymbol{\lambda}_{\mathrm{f}} + \boldsymbol{\lambda}_{\mathrm{vp}} + \boldsymbol{\lambda}_{\mathrm{app}}$

Instantaneous Dynamics Model

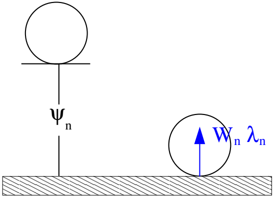
G = Jacobian of kinematic velocity map

- Newton-Euler Equations:
  $\mathbf{M}(\mathbf{q})\dot{\boldsymbol{\nu}} = \mathbf{W}_{\mathrm{n}}\boldsymbol{\lambda}_{\mathrm{n}} + \mathbf{W}_{\mathrm{f}}\boldsymbol{\lambda}_{\mathrm{f}} + \boldsymbol{\lambda}_{\mathrm{vp}} + \boldsymbol{\lambda}_{\mathrm{app}}$
- Kinematic Map: $\dot{\mathbf{q}} = G\boldsymbol{\nu}$

Instantaneous Dynamics Model

$\psi_{i\mathrm{n}}$ = signed distance function at contact i

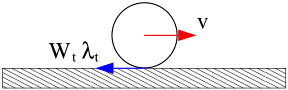
- Newton-Euler Equations:
  $\mathbf{M}(\mathbf{q})\dot{\boldsymbol{\nu}} = \mathbf{W}_\mathrm{n}\boldsymbol{\lambda}_\mathrm{n} + \mathbf{W}_\mathrm{f}\boldsymbol{\lambda}_\mathrm{f} + \boldsymbol{\lambda}_\mathrm{vp} + \boldsymbol{\lambda}_\mathrm{app}$
- Kinematic Map: $\dot{\mathbf{q}} = G\boldsymbol{\nu}$
- Normal Complementarity Constraint:
  $0 \leq \boldsymbol{\lambda}_{i\mathrm{n}} \perp \psi_{i\mathrm{n}}(\mathbf{q}, t) \geq 0$



Instantaneous Dynamics Model

$\mathbf{v}_i$ = relative velocity at contact i
$F_i(\boldsymbol{\lambda}_{in}, \mu)$ = friction cone at contact i

- Newton-Euler Equations: $\mathbf{M}(\mathbf{q})\dot{\boldsymbol{\nu}} = \mathbf{W}_{\rm n}\boldsymbol{\lambda}_{\rm n} + \mathbf{W}_{\rm f}\boldsymbol{\lambda}_{\rm f} + \boldsymbol{\lambda}_{\rm vp} + \boldsymbol{\lambda}_{\rm app}$
- Kinematic Map: $\dot{\mathbf{q}} = G\boldsymbol{\nu}$
- Normal Complementarity Constraint: $0 \leq \boldsymbol{\lambda}_{in} \perp \psi_{in}(\mathbf{q}, t) \geq 0$
- Max. Power Dissipation: $\boldsymbol{\lambda}_{if} \in \operatorname{argmax}\{-\mathbf{v}_{if}\boldsymbol{\lambda}'_{if} : \boldsymbol{\lambda}'_{if} \in F_i(\boldsymbol{\lambda}_{in}, \mu)\}$



Discrete Time Dynamics Model

$$\dot{\boldsymbol{\nu}} \approx (\boldsymbol{\nu}^{\ell+1} - \boldsymbol{\nu}^{\ell})/h \qquad \dot{\mathbf{q}} \approx (\mathbf{q}^{\ell+1} - \mathbf{q}^{\ell})/h$$

$$\begin{aligned}
\mathbf{M}\boldsymbol{\nu}^{\ell+1} &= \mathbf{M}\boldsymbol{\nu}^{\ell} + h(\mathbf{W}_{\mathrm{n}}\boldsymbol{\lambda}_{\mathrm{n}}^{\ell+1} + \mathbf{W}_{\mathrm{f}}\boldsymbol{\lambda}_{\mathrm{f}}^{\ell+1} + \boldsymbol{\lambda}_{\mathrm{vp}}^{\ell} + \boldsymbol{\lambda}_{\mathrm{app}}^{\ell}) \\
\mathbf{q}^{\ell+1} &= \mathbf{q}^{\ell} + hG\boldsymbol{\nu}^{\ell+1} \\
0 \leq h\boldsymbol{\lambda}_{\mathrm{n}}^{\ell+1} &\perp \psi_{\mathrm{n}}(\mathbf{q}^{\ell+\mathbf{1}}) \geq 0 \\
\boldsymbol{\lambda}_{i\mathrm{f}}^{\ell+1} &\in \operatorname{argmax}\{-\mathbf{v}_{i\mathrm{f}}^{\ell+1}\boldsymbol{\lambda}_{i\mathrm{f}}' : \boldsymbol{\lambda}_{i\mathrm{f}}'^{\ell+1} \in F_i(\boldsymbol{\lambda}_{i\mathrm{n}}^{\ell+1}, \mu)\}
\end{aligned}$$

Where h is the length of the time step and superscripts ℓ and $\ell+1$ denote values at the beginning and end of the ℓth time step.

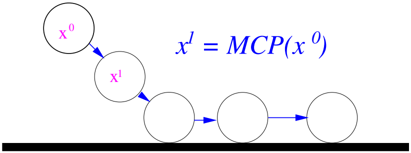
State Estimation

The dynamic system is modeled with two equations:

State Transition Equation

$$\mathbf{x}^{\ell+1} = F(\mathbf{x}^{\ell}, \mathbf{u}^{\ell}, \zeta^{\ell})$$

$$\mathbf{x} = [x_p \ y_p \ \dot{x}_p \ \dot{y}_p \ \lambda_\text{n} \ \lambda_\text{f} \ \sigma]$$



$F(\cdot)$: Dynamic model

$\mathbf{x}^{\ell}$: Unobserved state at time ℓ

$\mathbf{u}^{\ell}$: known input at time ℓ

ζ^{ℓ} : process noise at time ℓ

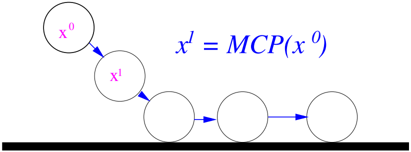
State Estimation

Measurement Equation

$$\mathbf{y}^{\ell} = H(\mathbf{x}^{\ell}, \mathbf{n}^{\ell})$$

$$\mathbf{y} = [x_p \ y_p]$$
$$\Leftrightarrow H(\mathbf{x}^{\ell}, \mathbf{n}^{\ell}) = [\mathbf{x}_1^{\ell} \ \mathbf{x}_2^{\ell}] + \mathbf{n}^{\ell}$$

$H(\cdot)$: Measurement Function
$\mathbf{x}^{\ell}$: Unobserved state at time ℓ
$\mathbf{y}^{\ell}$: Observed state at time ℓ
$\mathbf{n}^{\ell}$: Observation noise at time ℓ



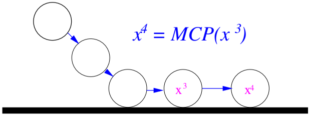
Parameter Estimation

Determine the nonlinear mapping:

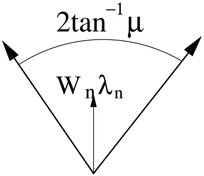
$$\mathbf{y}^{\ell} = G(\mathbf{x}^{\ell}, \mathbf{p})$$

$G(\cdot)$: Nonlinear Map

$\mathbf{p}$: Parameters of the mapping



Coulomb's law: $\lambda_{\mathsf{f}} \leq \mu \lambda_{\mathsf{n}}$

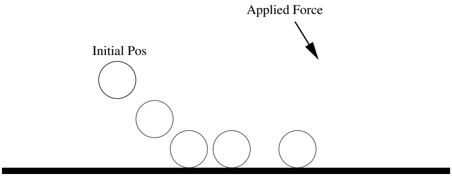


Dual Estimation

Special case where both the state and parameters must be learned simultaneously.

$$\mathbf{x}^{\ell+1} = F(\mathbf{x}^{\ell}, \mathbf{u}^{\ell}, \zeta^{\ell}, \mathbf{p})$$
$$\mathbf{y}^{\ell} = H(\mathbf{x}^{\ell}, \mathbf{u}^{\ell}, \mathbf{n}^{\ell}, \mathbf{p})$$

Both $\mathbf{x}^{\ell}$, $\ell = 1, 2, \ldots N$ and $\mathbf{p}$ must be simultaneously estimated.



$$\mathbf{x}^{\ell} = [x_p^{\ell} \; y_p^{\ell} \; \dot{x}_p^{\ell} \; \dot{y}_p^{\ell} \; \lambda_{\mathrm{n}}^{\ell} \; \lambda_{\mathrm{f}}^{\ell} \; \sigma^{\ell}]$$
$$\mathbf{y}^{\ell} = [x_p^{\ell} \; y_p^{\ell}]$$
$$\mathbf{p} = [\mu]$$
$$F(\cdot) = \text{Discrete time } MCP$$
$$H(\cdot) = [x_p^{\ell} \; y_p^{\ell}] + \mathbf{n}^{\ell}$$

Difficulties of Current Approaches

Difficulties of Kalman Filtering

- Not possible to apply physical constraints to parameters or state (e.g. $\mu > 0$)
- Noise is assumed to be Gaussian
- Fails with multimodal pdfs

Difficulties of Particle Filtering

- Difficult to apply physical constraints to parameters or state
- With small process noise, all particles can collapse into a single point within a few iterations

Problem Formulation

Optimization Problem for Dual Estimation of Rigid Body Dynamics

$$\min_{\mathbf{n}^0,\ldots,\mathbf{n}^N,\ \mathbf{x}^0,\ldots,\mathbf{x}^N,\ p} (\mathbf{x}^0 - \bar{\mathbf{x}}^0)^T(\mathbf{x}^0 - \bar{\mathbf{x}}^0) + \sum_{\ell=0}^{T} \mathbf{n}^{\ell^T}\mathbf{n}^\ell \quad (1)$$

$$\text{subject to:} \quad \mathbf{p} \in \mathcal{P}, \mathbf{n} \in \mathcal{N} \quad (2)$$

$$\mathbf{x}^{\ell+1} \in SOL(\text{MCP}(\mathbf{x}^\ell, \mathbf{p})) \quad (3)$$

$$\mathbf{y}^\ell = [\mathtt{I}\ \mathbf{0}]\mathbf{x}^\ell + \mathbf{n}^\ell \quad (4)$$

where $\bar{\mathbf{x}}^0$ is the initial state estimate, $\mathbf{n}$ is a slack variable representing the error between observation and prediction, $\mathtt{I}$ is an identity matrix of appropriate size, MCP is the mixed complementarity problem arising from the discrete time dynamics model, and $\mathcal{P}$ and $\mathcal{N}$ are the sets of possible parameter values and max observations error respectively.

MPEC Definition

Definition

$$\min_{u \in \mathbb{R}^{n_1}, v \in \mathbb{R}^{n_2}} f(\mathbf{u}, \mathbf{v}) \tag{5}$$

$$\text{subject to: } (\mathbf{u}, \mathbf{v}) \in Z, \text{ and} \tag{6}$$

$$\mathbf{v} \text{ solves } \mathrm{MCP}(g(\mathbf{u}, \cdot), \mathbf{B}), \tag{7}$$

where f is a desired objective function, $Z \subseteq \mathbb{R}^{n_1+n_2}$ is a nonempty closed set (equation (6) represents standard nonlinear programming constraints), and equation (7) signifies $\mathbf{v}$ is a solution to the MCP defined by the function g and the bound set $\mathbf{B}$.

For the special case when f and the MCP are linear, the problem is known as a linear program with equilibrium constraints (LPEC).

Restrictions of an LPEC

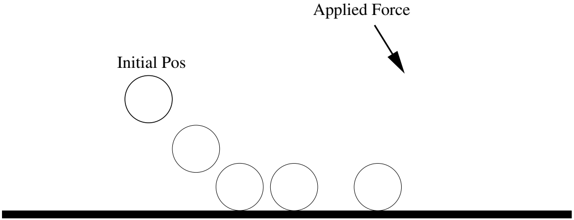
If we wish to remove the nonlinearties, we would be forced to make the following assumptions:

- Rotations are restricted to being “small” because the contact Jacobians are a function of the bodies’ states.
- We must assume that either the parameters or state values are known to eliminate the bilinear constraints. For example, the friction model contains the bilinear constraint: $\mu\lambda_{\mathsf{n}}^{\ell+1}$

Solution Techniques

We use the AMPL mathematical modeling language to formulate the MPECs, and the nonlinearly constrained optimization solvers available on the NEOS Server.

Simple Example Scene



Goal: Estimate the coefficient of friction (μ) from the "noisy" observations. Noise was added by uniformally sampling $[-\epsilon,\ \epsilon]$.

Estimation Formulation

$$\text{State: } \mathbf{x} = [\overbrace{x_p \; y_p \; \dot{x}_p \; \dot{y}_p}^{\text{Observable}} \mid \overbrace{\lambda_\mathrm{n} \; \lambda_\mathrm{f} \; \sigma}^{\text{Unobservable}}]$$

$$\text{Observations: } \mathbf{y} = [x_p \; y_p \; \dot{x}_p \; \dot{y}_p]$$

$$\text{Parameters: } \mathbf{p} = [\mu]$$

We cannot observe the velocity directly, but can approximate it.

$$\mathbf{x}^{\ell+1} \in SOL(\mathrm{MCP}(\mathbf{x}^\ell)) \quad \text{(Described on next slide)} \tag{8}$$

$$\mathbf{y}^\ell = \left[\mathrm{I}_{4\times 4} \; \mathbf{0}_{4\times 4}\right] \mathbf{x}^\ell + \mathbf{n}^\ell \tag{9}$$

$$\mathcal{P} = 0 \leq \mu \leq 1 \tag{10}$$

$$\mathcal{N} = \mathbf{n}^{\ell^T} \mathbf{n}^\ell \leq \epsilon^2 \tag{11}$$

Equations of Motion

$$0 = q^{\ell+1} - q^{\ell} - h\boldsymbol{\nu}^{\ell+1}$$

$$0 = M(\boldsymbol{\nu}^{\ell+1} - \boldsymbol{\nu}^{\ell}) - h(W_{\mathrm{n}}\lambda_{\mathrm{n}}^{\ell+1} + W_{\mathrm{f}}\lambda_{\mathrm{f}}^{\ell+1} + \lambda_{\mathrm{app}})$$

$$0 \le \lambda_{\mathrm{n}}^{\ell+1} \perp y^{\ell+1} \ge 0 \quad \} \quad \text{Normal contact model}$$

$$\left.\begin{array}{l} 0 \le \lambda_{\mathrm{f}}^{\ell+1} \perp E\sigma^{\ell+1} + W_{\mathrm{f}}^{\mathrm{T}}\boldsymbol{\nu}^{\ell+1} \ge 0 \\ 0 \le \sigma^{\ell+1} \perp \mu\lambda_{\mathrm{n}}^{\ell+1} - E^{\mathrm{T}}\lambda_{\mathrm{f}}^{\ell+1} \ge 0 \end{array}\right\} \quad \text{Friction Model}$$

where $q = \begin{bmatrix} x_p \\ y_p \end{bmatrix}$, $\boldsymbol{\nu} = \begin{bmatrix} \dot{x}_p \\ \dot{y}_p \end{bmatrix}$, $M = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}$, $W_{\mathrm{n}} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$,
$W_{\mathrm{f}} = \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}$, $E = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, and $\lambda_{\mathrm{app}} = \begin{bmatrix} x_{\mathrm{app}} \\ -mg \end{bmatrix}$

4 equations and 4 complementarity constraints per time step.

MPEC Formulation

$$\min_{\mathbf{x}^0,\ldots,\mathbf{x}^N} (\mathbf{x}^0 - \bar{\mathbf{x}}^0)^T(\mathbf{x}^0 - \bar{\mathbf{x}}^0) + \sum_{\ell=0}^{N} \mathbf{n}^{\ell^T}\mathbf{n}^{\ell}$$

subject to:

$$\left.\begin{aligned} 0 &= q^{\ell+1} - q^{\ell} - h\boldsymbol{\nu}^{\ell+1} \\ 0 &= M(\boldsymbol{\nu}^{\ell+1} - \boldsymbol{\nu}^{\ell}) - h(W_{\mathrm{n}}\lambda_{\mathrm{n}}^{\ell+1} + W_{\mathrm{f}}\lambda_{\mathrm{f}}^{\ell+1} + \lambda_{\mathrm{app}}) \end{aligned}\right\} \text{4N equations}$$

$$\left.\begin{aligned} 0 &\leq \lambda_{\mathrm{n}}^{\ell+1} \perp y^{\ell+1} \geq 0 \\ 0 &\leq \lambda_{\mathrm{f}}^{\ell+1} \perp E\sigma^{\ell+1} + W_{\mathrm{f}}^{\mathrm{T}}\boldsymbol{\nu}^{\ell+1} \geq 0 \\ 0 &\leq \sigma^{\ell+1} \perp \mu\lambda_{\mathrm{n}}^{\ell+1} - E^{\mathrm{T}}\lambda_{\mathrm{f}}^{\ell+1} \geq 0 \end{aligned}\right\} \text{4N complementarity}$$

$$\left.\begin{aligned} 0 &\leq \mu \leq 1 \\ \mathbf{n}^{\ell^T}\mathbf{n}^{\ell} &\leq \epsilon^2 \end{aligned}\right\} \text{4N + 1 inequalities}$$

$$\mathbf{y}^{\ell} = \left[\mathrm{I}_{4\times 4}\ \mathbf{0}_{4\times 4}\right]\mathbf{x}^{\ell} + \mathbf{n}^{\ell} \quad \} \quad \text{4N equations}$$

Results

Values used in simulation: $\mu = 0.2$, $h = .05$, $q^0 = [0, 3]$, $\boldsymbol{\lambda}_{app} = [5, -9.81m]$, $m = 1$, $N = 100$

Observe. Error	μ	Obj. Val	Iters
5.00E-05	0.2	2.03e-07	301
5.00E-04	0.2	1.61e-05	443
5.00E-03	0.2	1.467e-03	19
5.00E-02	0.200022	1.61e-01	149
5.00E-01	0.199873	1.48e+01	96 (infeasible pt)

Table: Solver: KNITRO

Initial guess for MPEC: $q^\ell = \tilde{q}^\ell$, $\lambda_{\mathrm{n}}^\ell = 0$, $\lambda_{\mathrm{f}}^\ell = 0$, $\sigma^\ell = 0$, $\mu = 1$.

Multiple Particles

We extend the previous example to now include multiple particles starting with random initial positions ($x \in [-10, 10]$, $y \in [0, 5]$) and random coefficients of friction ($\mu \in (0, 0.5]$).

Results

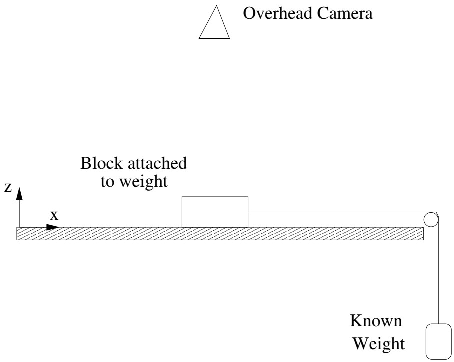
Measurement noise $\in [-0.005, 0.005]$.

# Part.	Num. Var.	μ Error	Obj. Val	Iters
2	2006	0	4.09e-03	334
3	3009	0	4.95e-03	185
5	5015	0.000006	1.05e-02	364
10	10030	0.0000072	1.66e-02	281

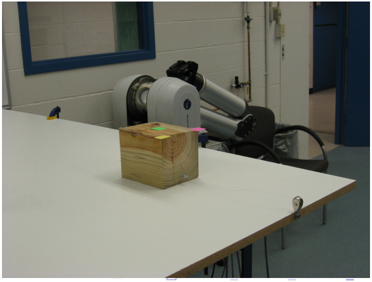
Table: Solver: KNITRO

μ error is the root mean squared error. Initial guess for MPEC: $q^\ell = \tilde{q}^\ell$, $\lambda_\mathrm{n}^\ell = 0$, $\lambda_\mathrm{f}^\ell = 0$, $\sigma^\ell = 0$, $\mu = 1$.

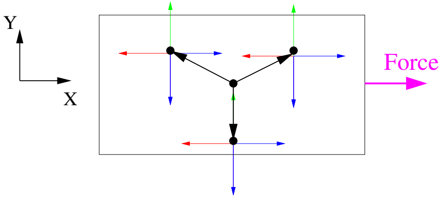
Sliding Block Scene



Experimental set up

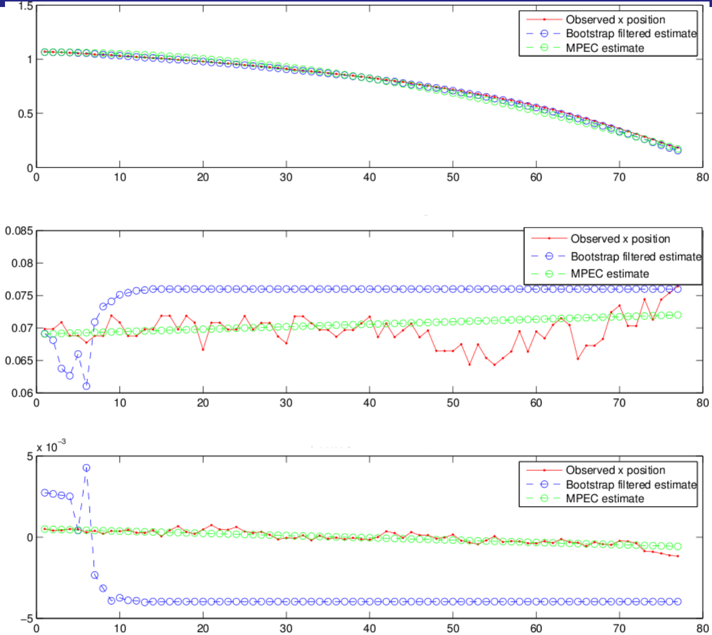


Goal

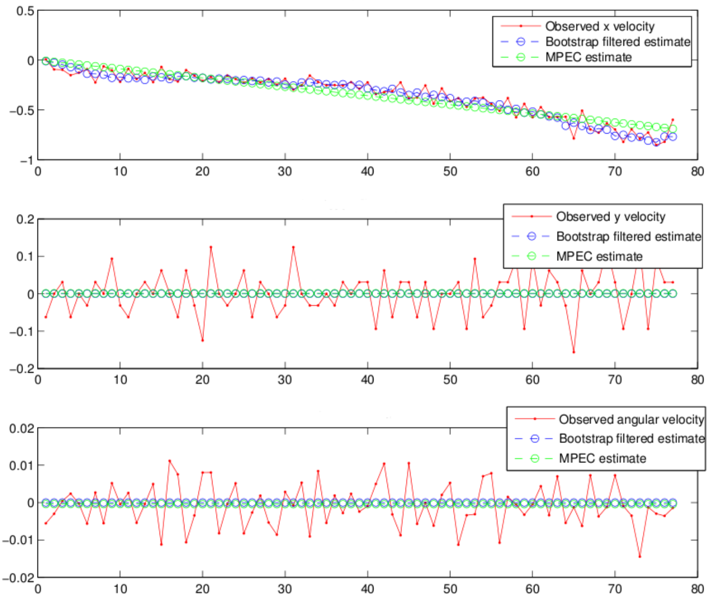


Surface friction support tripod

Goal is to determine the surface friction coefficient assuming a fixed support tripod with linearized friction cones.

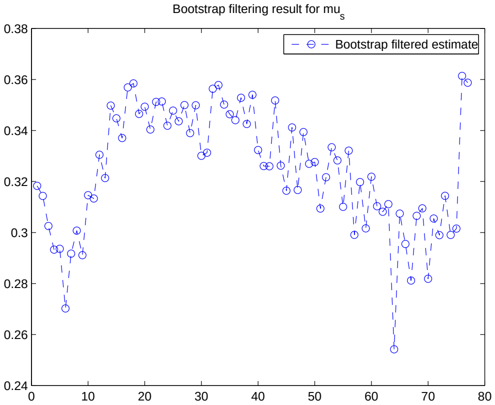


Comparison of trajectory between particle filter, MPEC and observation



Comparison of velocity between particle filter, MPEC and observation

Results



Surface friction estimate of the particle filter.

Mean of particle filter's estimate of $\mu = 0.3246$

MPEC estimate of $\mu = 0.330311$

Future Work

- Online Solution
  - Partial information
  - Moving horizon framework
- Observability of nonlinear and nonsmooth system

ACKNOWLEDGMENT

This work was supported by the National Science Foundation under grants 0413227 (IIS-RCV), and 0420703 (MRI)